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On some radiation pressure effects produced by wave packets in plasmas

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Abstract. An obliquely impinging electromagnetic wave exerts a negative radiation pressure on an abrupt plasma–vacuum boundary. The radiation pressure on a conductor immersed in plasma consists of an electromagnetic part and a mechanical part. The equation of motion of a quasi-particle corresponding to a wave packet is derived. The time-averaged motion of a charged particle in a non-monochromatic wave packet is given. No assumption is made concerning the validity of Abraham or Minkowski momentum densities.

1. Introduction

The recent growth of interest in radiation pressure effects has been motivated mostly by new experimental capabilities of laser devices. Another attractive point here is the well-known contradiction between the Abraham and Minkowski field momentum density in dielectrics (see Brevik 1970, Ginzburg 1973 and Skobel'tsyn 1973 for a review). In the case of the presence of oscillating fields the analysis may be more complicated due to the procedure of time averaging. The basic point, at least regarding the fluid dynamics, is the knowledge of an equation for the time-averaged fluid velocity. The general form of this equation for arbitrary media has hitherto not been derived.

As an exception to this situation, the classical plasma seems to be readily accessible for theoretical study. Recalling the reviews published by Motz and Watson (1967) and by Gorbunov (1973), one may observe considerable progress in understanding many radiation pressure effects in plasmas. An interesting consequence of the action of electrodynamical forces, namely, the expansion of a weakly inhomogeneous plasma, has been predicted by Hora *et al* (1967) (see also Hora 1969, Lindl and Kaw 1971). These expansive forces are closely connected with the mechanical momentum density (Klíma and Petržílka 1972)

$$p_z = \frac{1-\epsilon}{16\pi c\sqrt{\epsilon}} E_0(z-v_g t)E_0^*(z-v_g t), \tag{1.1}$$

which is transported in a one-dimensional quasi-monochromatic wave packet; in (1.1) E_0 is the electric field amplitude, $v_g = c\sqrt{\epsilon}$ is the group velocity, $\epsilon = 1-\omega_p^2/\Omega^2$, ω_p is the plasma frequency and Ω is some mean frequency of the wave packet. The presence of the momentum (1.1) also explains (Klíma and Petržílka 1972) the rather unexpected zero radiation pressure (Klíma and Petržílka 1968) at perpendicular incidence of an electromagnetic wave upon an abrupt plasma–vacuum boundary. The energy–momentum tensor including (1.1) implies (Klíma and Petržilka 1973c) simple particle-like properties of a classical wave packet and, moreover, the equivalence of Einstein's energy–mass relation to the wave dispersion law. The mechanical momentum transport also appears to be relevant for the radiation pressure effects in non-dispersive dielectrics (Skobel'tsyn 1973, Gordon 1973).

The purpose of the present paper is to develop the above theory of momentum transport by considering a few simple models. In § 2, the force exerted on an abrupt plasma–vacuum boundary by an obliquely propagating electromagnetic wave is derived. The evidence of radiation pressure on a conductor immersed in a liquid (Jones and Richards 1954) stimulated the authors to examine an analogous situation for a plasma (§ 3). Section 4 extends the particle-like properties of a wave packet mentioned above to the case of a weakly inhomogeneous plasma. While quasi-monochromatic waves are considered in §§ 2, 3, 4, a substantially non-monochromatic wave packet is studied in § 5.

2. Abrupt plasma–vacuum boundary

In Cartesian coordinates x, y, z, isotropic collisionless and cold plasma is assumed to be in the half-space $z > 0$. The refractive index is $N = 1$ for $z < 0$ and $N = \sqrt{\epsilon}$ for $z > 0$. The plane (x, z) is the plane of incidence of a quasi-monochromatic wave packet coming from vacuum. The corresponding electric field intensity $\boldsymbol{E}^{\mathrm{I}}$ varies only in the direction of propagation and lies in the (x, z) plane:

$$\boldsymbol{E}^{\mathrm{I}} = (\cos\alpha, 0, -\sin\alpha)E_0^{\mathrm{I}}(x, z, t)\exp\left(\mathrm{i}\frac{\Omega}{c}(x\sin\alpha + z\cos\alpha - ct)\right) \tag{2.1}$$

where E_0^{I} is the slowly varying amplitude,

$$E_0^{\mathrm{I}}(x, z, t) = \int_{-\infty}^{+\infty} \mathrm{d}\omega\, E(\omega)\exp\left(\mathrm{i}\frac{\omega-\Omega}{c}(x\sin\alpha + z\cos\alpha - ct)\right) \tag{2.2}$$

and α is the angle of incidence, α being less than $\sin^{-1}N$. By using the boundary conditions at $z = 0$,

$$E_0^{\mathrm{I}}\cos\alpha + E_0^{\mathrm{R}}\cos\alpha = E_0^{\mathrm{P}}\cos\beta, \qquad \sqrt{\epsilon}\sin\beta = \sin\alpha, \qquad E_0^{\mathrm{I}} - E_0^{\mathrm{R}} = E_0^{\mathrm{P}}\sqrt{\epsilon},$$

the intensities of the reflected and the transmitted waves are readily obtained in the following form:

$$\boldsymbol{E}^{\mathrm{R}} = (\cos\alpha, 0, \sin\alpha)E_0^{\mathrm{R}}(x, z, t)\exp\left(\mathrm{i}\frac{\Omega}{c}(x\sin\alpha - z\cos\alpha - ct)\right) \tag{2.3}$$

$$\boldsymbol{E}^{\mathrm{P}} = (\cos\beta, 0, -\sin\beta)E_0^{\mathrm{P}}(x, z, t)\exp\left(\mathrm{i}\frac{\Omega}{c}(x\sin\alpha + z\sqrt{\epsilon}\cos\beta - ct)\right) \tag{2.4}$$

$$E_0^{\mathrm{R}}(x, z, t) = \frac{\cos\beta - \sqrt{\epsilon}\cos\alpha}{\cos\beta + \sqrt{\epsilon}\cos\alpha}\int_{-\infty}^{+\infty} \mathrm{d}\omega\, E(\omega)\exp\left(\mathrm{i}\frac{\omega-\Omega}{c}(x\sin\alpha - z\cos\alpha - ct)\right) \tag{2.5}$$

$$E_0^{\mathrm{P}}(x, z, t) = \frac{2\cos\alpha}{\sqrt{\epsilon}\cos\alpha + \cos\beta}\int_{-\infty}^{+\infty} \mathrm{d}\omega\, E(\omega)\exp\left[\mathrm{i}\frac{\omega-\Omega}{c}\left(x\sin\alpha + \frac{z\cos^2\alpha}{\sqrt{\epsilon}\cos\beta} - ct\right)\right]. \tag{2.6}$$

To simplify the algebra it is useful to introduce variables

$$\xi = x\cos\kappa - z\sin\kappa$$

$$\eta = x\sin\kappa + z\cos\kappa$$

with

$$\sin\kappa = \frac{\sqrt{\epsilon}\sin\alpha\cos\beta}{[1+(\epsilon-2)\sin^2\alpha]^{1/2}}. \tag{2.7}$$

We note that $\kappa < \alpha < \beta$ at $\alpha \neq 0$. The amplitude of the transmitted wave $E_0^{\rm P}$ then depends only upon the argument $\eta - Vt$,

$$V = \frac{c\sqrt{\epsilon}\cos\beta}{[1+(\epsilon-2)\sin^2\alpha]^{1/2}}. \tag{2.8}$$

Consequently the envelope of the transmitted wave packet (figure 1) moves with the velocity $\boldsymbol{V} = (\sin\kappa, 0, \cos\kappa)V$. The group velocity $\boldsymbol{v}_{\rm g} = \partial\omega/\partial\boldsymbol{k} = (\sin\beta, 0, \cos\beta)c\sqrt{\epsilon}$ is related to $\boldsymbol{V}$ as

$$V = v_{\rm g}\cos(\beta - \kappa). \tag{2.9}$$

The difference between the group velocity $\boldsymbol{v}_{\rm g}$ and the envelope velocity $\boldsymbol{V}$ stems from the one-dimensionality of the wave packet. In other words the component of $\boldsymbol{v}_{\rm g}$ perpendicular to $\boldsymbol{V}$ can be ignored in the envelope motion.

The above information about the field quantities is sufficient for calculating the time-averaged forces. It has been shown by Fainberg and Shapiro (1965) that the time dependence of field amplitude may produce a force proportional to $\partial E^2/\partial t$. For a transverse wave, however, this force vanishes (cf Klíma 1972) and only the well-known

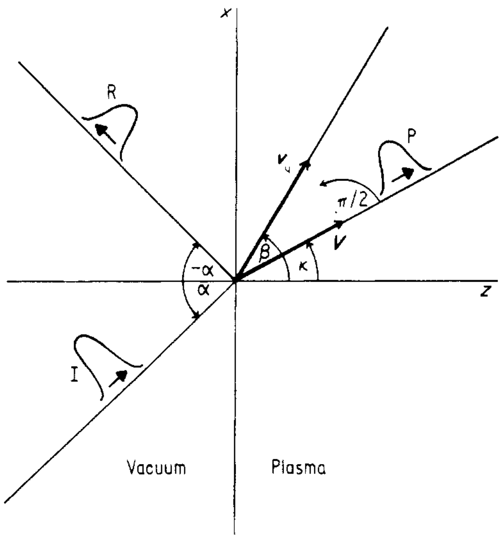

Figure 1. A schematic representation of the incident (I), reflected (R) and transmitted (P) wave packets. The angles of incidence and of reflection are α and $-\alpha$, respectively. The direction of the group velocity $\boldsymbol{v}_{\rm g}$ is determined by β, whereas κ is the angle between the z axis and the velocity $\boldsymbol{V}$ of the motion of the transmitted wave packet envelope.

force proportional to $(-\nabla E^2)$ remains. Consequently, the tensor of time-averaged stresses

$$T_{mn} = -\frac{1}{16\pi}\delta_{mn}(E_j^* E_j + H_j^* H_j) + \frac{1}{16\pi}(H_m^* H_n + H_m H_n^* + \epsilon_{mk} E_k E_n^* + \epsilon_{km} E_k^* E_n) \tag{2.10}$$

can be used in deriving the surface force density F^S which acts on the plasma–vacuum boundary:

$$F_x^S = T_{xz}(z \to 0+) - T_{xz}(z \to 0-) = 0, \qquad F_y^S = 0, \tag{2.11}$$

$$F_z^S = T_{zz}(z \to 0+) - T_{zz}(z \to 0-)$$
$$= -\frac{E_0^I E_0^{I*}}{16\pi} \frac{4(1-\epsilon)^2 \cos^2\alpha \sin^2\beta}{(\sqrt{\epsilon}\cos\alpha + \cos\beta)^2}. \tag{2.12}$$

It should be emphasized that the force density F^S is directed from plasma into vacuum. Physically, this force arises due to the presence of an oscillating surface charge, quite similarly as in the case of an electrostatic field. A somewhat different result has been derived for the case of an incompressible fluid with $\epsilon > 1$ (Kats and Kontorovich 1969), where, however, the electrostrictive force has been neglected.

To complete the physical picture, it is useful to express the relevant components of the stress tensor T_{mn} in terms of the momentum densities. The time-averaged momentum density p of plasma particles is given by the equation (see eg Klíma 1972)

$$\frac{\partial p}{\partial t} = -\frac{1-\epsilon}{16\pi}\nabla(E_0^P E_0^{P*}), \tag{2.13}$$

which is identical with

$$\frac{\partial p_x}{\partial t} = \frac{\partial T_{xx}}{\partial x} + \frac{\partial T_{xz}}{\partial z} - \frac{\partial g_x^P}{\partial t}, \tag{2.14}$$

$$\frac{\partial p_z}{\partial t} = \frac{\partial T_{xz}}{\partial x} + \frac{\partial T_{zz}}{\partial z} - \frac{\partial g_z^P}{\partial t}, \tag{2.15}$$

where

$$g^P = (\sin\beta, 0, \cos\beta)\frac{\sqrt{\epsilon}E_0^P E_0^{P*}}{8\pi c} \tag{2.16}$$

is the momentum density of the electromagnetic field. Integrating (2.13) and neglecting the terms of higher than second order in E_0^P, we have

$$p = (\sin\kappa, 0, \cos\kappa)\frac{1-\epsilon}{16\pi V}E_0^P(\eta - Vt)E_0^{P*}(\eta - Vt). \tag{2.17}$$

From (2.14) and (2.15) it is easy to find that

$$-T_{xx} = p_x V_x + g_x^P v_{gx}, \tag{2.18}$$

$$-T_{xz} = -T_{zx} = p_x V_z + g_x^P v_{gz} = p_z V_x + g_z^P v_{gx}, \tag{2.19}$$

$$-T_{zz} = p_z V_z + g_z^P v_{gz}. \tag{2.20}$$

It is obvious that the momentum density flux T_{mn} is the sum of the following two parts:

(i) the mechanical momentum flux $p_m V_n$,

(ii) the electromagnetic momentum flux $g_m^P v_{gn}$.

Substituting (2.20) and the corresponding free space quantities into (2.12) we obtain

$$F_z^{\mathrm{S}} = (g_z^{\mathrm{I}} - g_z^{\mathrm{R}})c \cos\alpha - p_z V_z - g_z^{\mathrm{P}} v_{gz}, \tag{2.21}$$

which is nothing more than the momentum conservation law. For $\alpha = 0$ the zero surface force (Klíma and Petržílka 1968, 1972) is re-derived.

In addition to the surface force $\boldsymbol{F}^{\mathrm{S}}$, a volume force of the order $(-\nabla E^2)$ arises due to the given space dependence of the field amplitude in the incident wave packet. Consequently plasma is accelerated in front of the wave packet maximum and decelerated behind the maximum, which is just the origin of the mechanical momentum $\boldsymbol{p}$. This plasma motion leads to the following perturbation of particle concentration (Klíma and Petržílka 1972):

$$\delta n = \frac{1-\epsilon}{16\pi m_{\mathrm{i}} V^2} E_0^{\mathrm{P}} E_0^{\mathrm{P}*}, \tag{2.22}$$

where m_{i} is the ion mass. It has been shown recently that the corresponding rest mass density $m_{\mathrm{i}} c^2 \delta n$ is a relevant part of the relativistic energy–momentum tensor (Klíma and Petržílka 1973a, b). Introducing the total energy density transport

$$\boldsymbol{S}^{\mathrm{tot}} = c^2 \boldsymbol{g}^{\mathrm{P}} + m_{\mathrm{i}} c^2 \boldsymbol{V} \delta n,$$

we obtain the necessary relativistic relation (Landau and Lifshitz 1962),

$$\boldsymbol{S}^{\mathrm{tot}} = c^2(\boldsymbol{g}^{\mathrm{P}} + \boldsymbol{p}) = c^2 \boldsymbol{p}^{\mathrm{tot}}. \tag{2.23}$$

As to the wave polarized perpendicularly to the plane of incidence, the surface force density is zero.

3. Plasma–perfect conductor boundary

Consider the reflection of the wave packet impinging perpendicularly on the surface $z = 0$ of a perfect conductor. The field of this wave packet is given by equations (2.4) and (2.6) with $\alpha = \beta = 0$, the field of the reflected wave being $-\boldsymbol{E}^{\mathrm{P}}(-z, t)$. The force density $\boldsymbol{F}^{\mathrm{C}}$ acting upon the conductor surface is found immediately by using the stress tensor (2.10), the boundary condition $\boldsymbol{E}(z = 0) = 0$ and the relation $H_0^{\mathrm{P}} H_0^{\mathrm{P}*} = \epsilon E_0^{\mathrm{P}} E_0^{\mathrm{P}*}$:

$$F_z^{\mathrm{C}} = -T_{zz}(z \to 0-) = \frac{\epsilon}{4\pi}(E_0^{\mathrm{P}} E_0^{\mathrm{P}*})_{z=0} = 2 v_{\mathrm{g}} g^{\mathrm{P}}(z = 0), \tag{3.1}$$

where g^{P} is the magnitude of the field momentum density (2.16) of the incident wave. Thus the impulse (per cm^2, which will not be repeated below) given to the conductor during the entire reflection process is

$$P^{\mathrm{C}} = 2 v_{\mathrm{g}} \int_{-\infty}^{+\infty} g^{\mathrm{P}}(z = 0)\, \mathrm{d}t = 2G^{\mathrm{P}}, \tag{3.2}$$

where G^{P} is the electromagnetic field momentum of the whole incident wave packet:

$$G^{\mathrm{P}} = \frac{\sqrt{\epsilon}}{8\pi c} \int E_0^{\mathrm{P}} E_0^{\mathrm{P}*}\, \mathrm{d}z. \tag{3.3}$$

Naturally, the problem arises why the mechanical momentum discussed in § 2 does not enter relation (3.2). To answer this question let us determine (from (2.13)) the mechanical momentum density left at a point z by the incident and the reflected wave packets:

$$p^{\mathrm{L}}(z)=\frac{1-\epsilon}{16\pi}\int_{-\infty}^{+\infty}\mathrm{d}t\,\frac{\partial}{\partial z}\left[E_0^{\mathrm{P}}(z-v_{\mathrm{g}}t)E_0^{\mathrm{P}*}(-z-v_{\mathrm{g}}t)\exp\left(2\mathrm{i}\frac{\Omega}{c}z\sqrt{\epsilon}\right)+\mathrm{cc}\right],\tag{3.4}$$

where cc stands for complex conjugate. If the wave packet length is l the value of $p^{\mathrm{L}}(z)$ obviously differs from zero only in the layer $(-l/2)<z<0$. The whole mechanical momentum in this layer is

$$P^{\mathrm{L}}=\int_{-l/2}^{0}p^{\mathrm{L}}(z)\,\mathrm{d}z=2P,\tag{3.5}$$

where

$$P=\frac{1-\epsilon}{16\pi c\sqrt{\epsilon}}\int_{(l)}E_0^{\mathrm{P}}E_0^{\mathrm{P}*}\,\mathrm{d}z\tag{3.6}$$

is the mechanical momentum of the whole incident wave packet (see also Klíma and Petržílka 1972). Consequently the momentum conservation law

$$P+G^{\mathrm{P}}=P^{\mathrm{L}}+P^{\mathrm{C}}-P-G^{\mathrm{P}}\tag{3.7}$$

is satisfied just due to the presence of the momentum P^{L}. The left side of (3.7) belongs to the incident wave and the last two terms of the right-hand side belong to the reflected wave.

To determine the detailed distribution of P^{L} along the z axis one has to choose some explicit expression for the wave packet envelope, eg

$$E_0^{\mathrm{P}}=A\exp(-\xi^2/2a^2),\tag{3.8}$$

where A, a are constants and $\xi=z-v_{\mathrm{g}}t$. Using (3.4) we have

$$p^{\mathrm{L}}(z)=\frac{4P}{a}\left[\frac{a}{c}\Omega\sqrt{\epsilon}\sin\left(2\frac{\Omega}{c}z\sqrt{\epsilon}\right)+\frac{z}{a}\cos\left(2\frac{\Omega}{c}z\sqrt{\epsilon}\right)\right]\exp\left(-\frac{z^2}{a^2}\right).\tag{3.9}$$

The quasi-periodic structure of this expression arises from the fact that, during the process of reflection, a quasi-standing wave is formed which acts on plasma by the force of the order $(-\nabla E^2)$.

Summarizing the results one can conclude that the momentum $2G^{\mathrm{P}}$ is transferred to the conductor immediately during the wave packet reflection. An additional pressure arises due to the mechanical momentum density p^{L} of plasma particles. Since the transfer of this momentum may be rather slow, relaxation processes should be included in the corresponding theoretical description which, however, is not known.

4. Meshtserski equation for a wave packet

Let us now consider a quasi-monochromatic wave packet moving along the z axis in a slightly inhomogeneous plasma with $\epsilon=\epsilon(z)>0$. The validity of the first WKB approximation is assumed so that the field amplitude is proportional to $\epsilon^{-1/4}$. If the length l of the wave packet is much less than $(\mathrm{d}\ln\epsilon/\mathrm{d}z)^{-1}$, the following energy–mass relation holds (Klíma and Petržílka 1973c):

$$(Mc^2)^2=(P+G^{\mathrm{P}})^2c^2+(M_{\mathrm{g}}c^2)^2,\tag{4.1}$$

where

$$M = \frac{1+\epsilon}{16\pi c^2 \epsilon} \int_{(l)} E_0^{\mathrm{P}} E_0^{\mathrm{P}*}\, \mathrm{d}z = \frac{P+G^{\mathrm{P}}}{v_{\mathrm{g}}} \tag{4.2}$$

is the mass of the wave packet and

$$M_{\mathrm{g}} = M\left(1-\frac{v_{\mathrm{g}}^2}{c^2}\right)^{1/2}$$

is its rest value. We intend to show that the wave packet in question moves as a particle with a variable rest mass.

Equation (4.1) implies that

$$c^2 \frac{\mathrm{d}M}{\mathrm{d}t} = Fv_{\mathrm{g}} + c^2 \frac{M_{\mathrm{g}}}{M} \frac{\mathrm{d}M_{\mathrm{g}}}{\mathrm{d}t}, \tag{4.3}$$

where $F = (\mathrm{d}/\mathrm{d}t)(P+G^{\mathrm{P}})$. Using (3.3) and (3.6) it is easy to verify that F is the well-known force acting on the inhomogeneous plasma (Hora 1969). In other words the momentum lost or gained by the wave packet is transferred to or from the plasma inhomogeneity, respectively.

Supposing for a moment that $v_{\mathrm{g}} \ll c$, one obtains from (4.3) that

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}(M_{\mathrm{g}} v_{\mathrm{g}}^2) = v_{\mathrm{g}} \frac{\mathrm{d}}{\mathrm{d}t}(M_{\mathrm{g}} v_{\mathrm{g}}) - \frac{v_{\mathrm{g}}^2}{2}\frac{\mathrm{d}M_{\mathrm{g}}}{\mathrm{d}t}. \tag{4.4}$$

This is the Meshtserski equation for a particle with a variable mass in the case where the mass lost (or gained) by the particle moves (or originally moved) with zero velocity.

5. 'Blue-red' wave packet

The impulses of radiation coming from pulsars exhibit a characteristic time delay of the low-frequency part of their spectrum (eg ter Haar 1972). This delay arises due to the frequency dependence of the group velocity. Let us consider the time-averaged motion of a particle in such a substantially non-monochromatic wave packet. An idealized model of the wave packet is adopted, namely, we suppose that the instantaneous frequency Ω varies continuously along the wave packet (figure 2). The time-averaged velocity of a particle (charge e, mass m) is governed by the following equation (Klíma 1972):

$$\frac{\mathrm{d}V_z}{\mathrm{d}t} = -\frac{e^2}{2m^2\Omega^2}\left[\frac{1}{2}\frac{\partial E_0 E_0^*}{\partial z} + \frac{\partial k}{\partial t}\frac{E_0 E_0^*}{\Omega}\left(1+\frac{kV_z}{\Omega}\right) + \frac{\partial \Omega}{\partial z}\frac{kV_z}{\Omega^2} E_0 E_0^*\right], \tag{5.1}$$

where E_0 is the local field amplitude and $k = (\Omega/c)\sqrt{\epsilon}$ is the local wavenumber. The terms V_z^2/c^2 have been omitted in (5.1). In the interstellar space the inequality $1-\sqrt{\epsilon} \ll 1$ is well-satisfied, which simplifies the equation (5.1) considerably. Since, approximately,

$$\frac{\partial}{\partial t} + c\frac{\partial}{\partial z} = 0, \qquad \frac{\partial \Omega}{\partial z} = -\frac{\partial k}{\partial t}, \tag{5.2}$$

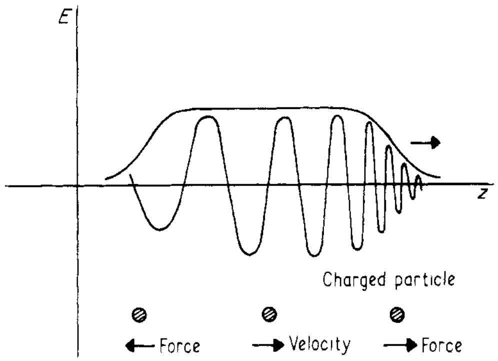

Figure 2. Electric field E of a wave packet with continuously varying frequency. The force on a charged particle is illustrated.

we have from (5.1)

$$\frac{dV_z}{dt} = \frac{e^2}{4m^2c^3}\frac{\partial}{\partial t}\frac{E_0E_0^*}{k^2}. \tag{5.3}$$

Neglecting again the terms of higher than second order in E_0, we obtain

$$\frac{dV_z}{dt} = \frac{e^2}{4m^2c^2(c-V_0)}\frac{d}{dt}\frac{E_0E_0^*}{k^2}, \tag{5.4}$$

where V_0 is the unperturbed particle velocity along the z axis. Consequently the total change of the particle momentum during the interaction with the whole wave packet is zero.

6. Conclusion

The surface force has been derived which is exerted on an abrupt plasma–vacuum boundary by an obliquely impinging electromagnetic wave. This force is perpendicular to the boundary and directed from plasma into vacuum. The corresponding stress tensor has been expressed in terms of electromagnetic and mechanical momentum densities.

The radiation pressure on a conductor immersed in plasma consists of two parts. While the first part is transferred to the conductor immediately during the reflection process, the second one is transported by plasma particles.

The particle-like description of a wave packet (Klíma and Petržilka 1973c) is extended by giving the equation of motion of the wave packet. At $v_g \ll c$ this equation is identical with the Meshtserski equation for a particle with varying mass.

The results of § 5 demonstrate a violation of the (otherwise rather universal) validity of the well-known ∇E^2-force. If the terms proportional to $\partial k/\partial t$, $\partial\Omega/\partial z$ were omitted in the equation of motion (5.1), a continual momentum transfer to particles would occur.

It should be emphasized that, throughout the paper, no assumption has been made about the validity of Abraham or Minkowski momentum densities.

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